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C. Pierre
Associate Editor

Predicting Ballistic Penetration and the Ballistic Limit in Composite Material Structures

Jeffrey R. Focht* and Jack R. Vinson†
University of Delaware, Newark, Delaware 19716

Introduction

THE ability to predict whether a given ballistic blunt object will penetrate a composite material structure is valuable. If penetration occurs the ability to predict the residual velocity of the object is also valuable. Also of interest is the ability to predict the ballistic limit of any projectile–composite structure combination.

Much work has been done in the field of analyzing the ballistic impact of composite targets; however, most of that work has been empirical in nature. Vinson and Zukas[1] introduced the use of conical shell theory as an analytical tool for determining the ballistic properties of woven Kevlar® targets. Then Vinson and Walker tested this model on fiber-reinforced composite targets.[2] The link between woven fabrics and composites was the assumption that if a matrix material contributed an insignificant amount to a fiber-reinforced composite's strength and strain properties then the entire structure would behave similarly to a woven fabric.

Vinson and Walker[2] utilized Lee and Sun's data[3] to test this analytical method and also compared the results with Ref. 3. At that time, the results seemed conclusive, but subsequent review of Ref. 2 has revealed that, although the underlying theory was correct, a computational error led to incorrect numerical results. This Note offers an improved method for determining the ballistic limit and revalidates the conical shell method for predicting if penetration will occur or not and if so what the residual velocity will be. In addition to validating the method against the data utilized in Ref. 2, more experiments were analyzed using data from other authors including Sun and Potti[4] and Silva et al.[5] The findings from these sources contribute to the validity of the model and also lead to additional conclusions about the conical shell method.

Analysis Methods

When a ballistic projectile impacts a composite plate, as shown in Fig. 1 in Ref. 2, a conical shell forms and continues to propagate until either the projectile velocity is reduced to zero or until the projectile penetrates the target material. Vinson and Walker[2] have shown that the conical shell is primarily in a state of membrane stress and that the resistance to penetration is mostly due to membrane strain energy. At the front of the conical shell is a radius R_1, which is determined by the dimensions of the blunt projectile.

The base radius of the shell is determined from the initial radius along with the propagation of a shear wave over time and is given as $R_2 = R_1(t = 0) + C_s t$. In this equation, C_s is the shear wave velocity taken to be $\sqrt{(G_{yz}/\rho_m)}$, where y is the meridional coordinate of the shell and z is the coordinate in the shell thickness direction. G_{yz} is the dynamic modulus at the given strain rate occurring; however, if that is not available, then the static value should be used.

Vinson and Walker[2] describe an iterative method to calculate the conical shell parameters and relate them to the ability of a composite target to capture a blunt projectile. When the required material properties and a striking velocity are supplied to an iterative solver, a time history is created that details the strain and projectile velocity at each time step. Test data consisting of striking velocity and residual velocity data points can then be used to correlate striking velocity to the ultimate strain at failure. This is accomplished by entering a given striking velocity and finding the time step that has a residual velocity that matches the experimental data point. The strain in the conical shell at that time step is determined to be the ultimate strain for that data point. The strain-striking velocity points are then plotted, and a linear regression is performed to determine the equation of the line of best fit for the data. This equation, then, relates striking velocity to maximum shell strain at failure. Examples of this are shown in Fig. 1.

The method used to determine the ballistic limit differs from that found in Ref. 2 and is considered the valid method for applying

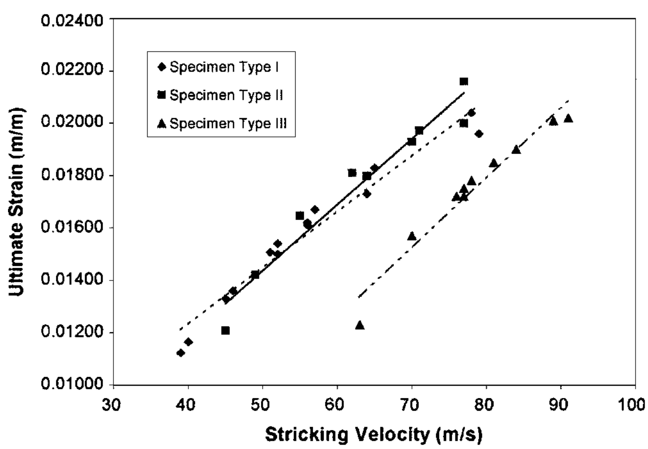

Fig. 1 Maximum strain to failure as a function of striking velocity for data of Ref. 3.

Received 16 October 2000; revision received 13 May 2002; accepted for publication 6 June 2002.

*Senior Engineering Student, Department of Mechanical and Aerospace Engineering; currently M.Me. Candidate, Mechanical, Aerospace and Nuclear Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180.

†H. Fletcher Brown Professor of Mechanical and Aerospace Engineering, Center for Composite Materials and College of Marine Studies. Fellow AIAA.

conical shell theory to the ballistic limit determination for fiber-reinforced composite targets. The calculated time history of the shell strain shows that the shell strain rises, reaches a maximum value, and then decreases. The ballistic limit is determined by varying striking velocity until the maximum strain given by the iterative solver matches the ultimate shell strain predicted by the linear regression analysis just detailed. The example to follow is shown to clarify the method.

Experimental Verification

Data for blunt ended projectiles have been taken from Lee and Sun,[3] Sun and Potti,[4] and Silva et al.[5] In addition, experimental data for rounded and pointed projectiles has been taken from Jenq, et al.[6] and Goldsmith, et al.[7] The purpose of this is to validate the conical shell analytical tool. The projectile and target properties needed for this method are enumerated in each respective reference. For purposes of completeness, the ballistic limit determined for each trial is compared to the ballistic limits calculated using the energy method in Ref. 3. The following example illustrates the process.

Example: Specimen Types 1–3 from Reference 3

This example is selected to compare the results with those in Ref. 2. The target material is Hercules AS4/3501-6 graphite epoxy, with a stacking sequence of $[0/90/45/-45]_s$. The projectile is blunt ended and made from hardened 4340 steel. The projectile has a radius of 7.3 mm and has a mass of 30.45 g. The properties of the target material can be found in Ref. 3.

For tests 1 and 2, the target material consists of 16 plies, and for test 3, the target consists of 32 plies. For this analysis the time steps are 5 μs. It can be shown that when a striking velocity of 45 m/s is input into the iterative model along with the correct target and projectile properties, the ultimate strain corresponding to a residual velocity of 9 m/s is 12.1×10^{-3} m/m. When this procedure is carried out for all of the striking velocity and residual velocity pairs for the Lee–Sun[3] data, a graph of striking velocity vs ultimate strain is generated and shown in Fig. 1.

When Fig. 1 is compared to Fig. 2 in Ref. 2, Fig. 1 confirms a strong linear relationship between striking velocity and ultimate strain for the corrected data, but several differences are observed. Ultimate strain values increase by nearly an order of magnitude from the results posted in Ref. 2, and these ultimate strains better correlate to the static strain limits for this material. The constant of proportionality between striking velocity and ultimate strain increases by a factor of two. Equation (1) summarizes the relationship between striking velocity and ultimate strain for the Lee–Sun[3] data.

Test specimen 1:

$$\varepsilon_{yf} = 3.726 \times 10^{-3} + 2.150 \times 10^{-4} V_s, \qquad R^2 = 0.958 \quad (1a)$$

Test specimen 2:

$$\varepsilon_{yf} = 1.782 \times 10^{-3} + 2.516 \times 10^{-4} V_s, \qquad R^2 = 0.947 \quad (1b)$$

Test specimen 3:

$$\varepsilon_{yf} = -3.620 \times 10^{-3} + 2.700 \times 10^{-4} V_s, \qquad R^2 = 0.998 \quad (1c)$$

From these experiments two more points must be made. First, the time required for the projectile to penetrate the target for the example enumerated is on the order of 1×10^{-3} s. This shows an increase in the length of the impact event as calculated by the conical shell model when compared to Ref. 2, which lists durations on the order of 1×10^{-5} s. Contrary to the findings of Ref. 2 with its computational error, the predicted time frame extends into the length of time where natural frequencies could be excited. However, neglecting natural frequency effects has yielded acceptable results. Additionally, because the time frame has increased compared to Ref. 2 results, the base radius of the conical shell extends beyond the natural boundaries of the targets, indicating the presence of wave reflection. The new results indicate that the presence of natural boundary conditions and wave reflection remains negligible when compared with the outgoing shear wave. These results correlate well against the test data, and Sun and Potti support this finding in Ref. 4.

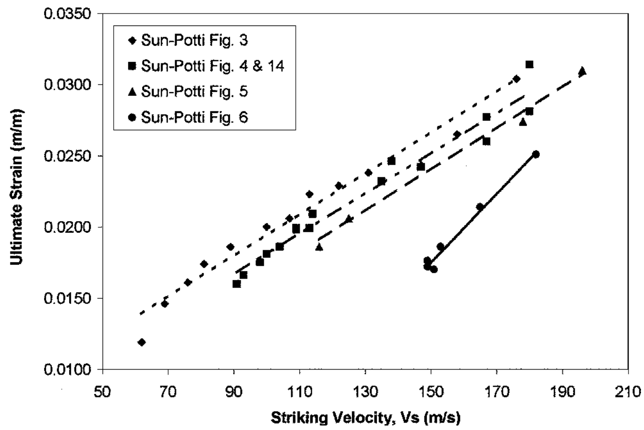

Fig. 2 Maximum strain to failure as a function of striking velocity for data of Refs. 4 and 5.

Additional Experiments

Data from Refs. 4 and 5 are examined to further verify the conical shell method. Figure 2 shows the ultimate strain vs striking velocity results for the Sun–Potti[4] data. Figure 2 demonstrates the linearity between strain and striking velocity for these blunt-ended projectile tests. Just as with the Lee–Sun[3] data, as the target thickness increases for a given material system, the strain function simply shifts downward with little change in slope. This shift predicts lower strain to failure for thicker targets, but maintains the same constant of proportionality between striking velocity and the ultimate strain for a given material system. Although this result is not immediately apparent, it is valuable for design purposes. This conclusion could be used to reduce the number of tests needed for a material system by interpolating target thicknesses that lie between tested target thicknesses as long as the stacking sequence is preserved.

In addition to blunt-ended projectiles, rounded and pointed projectiles were examined using the conical shell model with the data found in Refs. 6 and 7. This was to see if the theory and procedures of Refs. 1 and 2 would still apply. None of the cases provided usable results for the following two reasons. For some of the cases, the calculated residual velocity never reduced to the experimentally determined residual velocity so that ultimate strain data points could not be collected. In other cases, the ultimate strain vs striking velocity graph possessed a negative slope, and the resulting ballistic limit determinations did not make physical sense. These results confirm that projectile shape, that is, pointed or rounded, affects ballistic impact failures for composite targets, and this does not correlate with the conical shell predictions.

Ballistic Limit Determination

As stated earlier, in addition to providing a means to calculate the ultimate strain as a function of striking velocity for a given material system and predict residual velocities if penetration occurs, the conical shell model also can be employed to find the ballistic limit for blunt-ended projectiles. The ballistic limit for a particular target material is defined as the striking velocity where 50% of projectiles penetrate the target and 50% do not penetrate the target. This velocity is often called the V_{50}.

For each of the data sets detailed in the preceding section, the ballistic limit has been determined. The method prescribed in Ref. 3 for determining the ballistic limit was used to make the same determinations, and the two ballistic limit determinations were compared. Table 1 contains all of these ballistic limit determinations. Both determinations yield similar results, and both cases yield results that were higher than the experimental determinations.

The work shown here demonstrates a relatively simple way to determine the approximate ballistic limit with a minimum number of tests. Currently determining the ballistic limit of a target material

Table 1 Ballistic limit determinations

Reference	Conical shell method, V_{50}, m/s	Lee–Sun method,[3] V_{50}, m/s
Lee–Sun,[3] specimen 1	46	42
Lee–Sun,[3] specimen 2	42	47
Lee–Sun,[3] specimen 3	81	67
Sun–Potti,[4] Fig. 3	72	68
Sun–Potti,[4] Figs. 4 and 14	100	89
Sun–Potti,[4] Fig. 5	119	107
Sun–Potti,[4] Fig. 6	No limit found	146
Silva et al.[5]	126	192
Jenq et al.[6]	No limit found	159
Goldsmith et al.[7]	No limit found	23

requires a significant number of tests for each target thickness and requires rigorous statistical analysis. The conical shell method, on the other hand, requires only enough tests to determine accurately the ultimate strain as a function of striking velocity, and the results can be easily manipulated to generate ballistic limit determinations for targets with differing thickness.

Conclusions

The conical shell model of a ballistic impact provides a relatively simple tool to determine 1) the ultimate strain as a function of striking velocity, 2) whether penetration of a structure will occur, 3) the residual velocity of the projectile if penetration occurs, and 4) the ballistic limit for that target and projectile combination. This process serves to enhance the work found in Ref. 2 by offering an updated and validated method.

Another conclusion is that for a given material system the relationship between striking velocity and ultimate shell strain is linear where the slope is constant across targets of the same material and stacking sequence with different thickness. When the thickness of a target increases, the strain function shifts vertically in a graphical sense. This is important because it permits target thickness to be a variable subject to interpolation and reduces required testing.

Finally, this method provides a valuable analytical tool for the design and analysis of fiber/matrix composite material systems subject to ballistic impact because very few experiments are needed to provide this predictive capability. The methods are useful in that only a very limited number of tests are needed to establish the failure strain as a function of the striking velocity for a given target structure. Then those methods can be used to predict whether any other blunt ballistic object of known mass, size, and striking velocity will penetrate that structure and, if it does, what the residual velocity will be. The methods also provide the ballistic limit for other projectile sizes and the given composite target.

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S. Saigal
Associate Editor

Modified Short Beam Shear Test for Measuring Interlaminar Shear Strength of Composites

Kunigal Shivakumar,* Felix Abali,† and Adrian Pora‡
North Carolina A&T State University, Greensboro, North Carolina 27411

Introduction

INTERLAMINAR shear strength (ILSS) is an important material property for the design of laminated composite structures subjected to transverse loads. The American Society for Testing and Materials (ASTM)[1] proposed two test standards: three-point short beam shear (SBS) (D2344) and double-notched shear (D3846-99) tests to measure ILSS of composites. The four-point short beam shear (4P-SBS) test (ASTM D790) was also used in the literature. A number of analytical and experimental studies[2–7] have been conducted to determine the validity of these tests. Analytical studies include classical anisotropic beam and laminate analyses[2,6] and linear and nonlinear finite element analyses.[3–5] These studies concluded that the SBS test gives a qualitative estimate of ILSS and that many times failure is caused by indentation and/or flexure. A major problem of this test is the indentation deformation of the loading head. The waviness of textile fibers reduces the compression strength,[8] thus causing a compression failure on the loading side. Brittle matrix composites, like carbon-carbon composites, could also be crushed under the loading head. The 4P-SBS is an alternative test, but the loading heads arrangement restricts the specimen size.

Experimental study on the effect of indentation size[4,7] on failure of the SBS test showed that larger size indentation can increase the contact area and potentially reduce the indentation failure. Limitations of this hypothesis can be explained by the Hertzian contact analysis[9] of a steel cylinder pressing on a composite beam (insert in Fig. 1) resting on rigid support. Contact length c increased with increased cylinder radius and load q. Even with a 50-mm-radius cylinder the maximum half-contact length c that can be achieved is less than $\frac{1}{4}$ of the beam thickness, which is not enough to mitigate indentation failure.

Rahhal and Kotlesnsky[10] proposed a sandwiched specimen for carbon-carbon composite laminates and found some success. But sandwiching alters the material constraint and requires additional processing effort. Recently, Abali et al.[11] suggested a modification to the SBS test that alleviates the problems just mentioned. In this modified short beam shear (MSBS) test the central point loading is distributed uniformly over the middle half-span of the beam. This was achieved by the use of two pads (one stiff and the other soft) between the loading head and the specimen. This Note presents a summary of the analysis and test data for various textile fiber laminated polymer as well as carbon matrix composites.

Specimen Configuration and Loading

Figure 2 describes the specimen configuration and loading. The span is S, thickness t, width b, and the overhang length (or edge distance) t. The S/t ratio is proposed to be five for thin specimens

Received 9 April 2002; revision received 1 July 2002; accepted for publication 9 July 2002.

*Director, Center for Composite Materials Research; kunigal@ncat.edu. Associate Fellow AIAA.

†Adjunct Research Assistant Professor, Center for Composite Materials Research. Member AIAA.

‡Graduate Student, Center for Composite Materials Research. Member AIAA.